THE MOTION OF A DOUBLE WEDGE-SHAPED PROFILE AT A SPEED NOT EXCEEDING THAT OF SOUND

(O DVIZHENII ROMBOVIDNOGO PROFILIA SO SKOROST'IU NE PREVYSHAIUSHCHEI SKOROST' ZVUKA)

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1. Statement of problem. Consider the motion of a stream of gas with uniform subsonic velocity and at zero angle of attack past a double wedge-shaped profile (Fig.1), of length $2l$ and apex angle $2\pi/\xi = 2\delta$. Since the flow is symmetrical only the upper half-plane ($y > 0$) need be considered. The oncoming stream begins to slow down ahead of the profile, from some given velocity r_0 at infinity down to zero velocity at the apex O. After this, it speeds up again to reach sonic velocity along a parabolic line AB. This line is of finite length in the subsonic case, extends to infinity in a direction transverse to the flow in the sonic case, and in the supersonic cases it is terminated by means of at least one shock wave, the exact character of which is unknown. In sonic flow the hyperbolic region influences [1] only that part of the elliptical region, directly adjacent to the line of transition and to the limiting characteristic $B'F$.

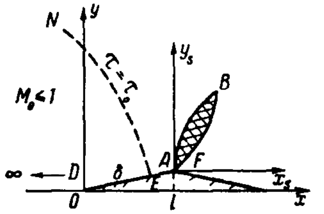

Fig. 1.

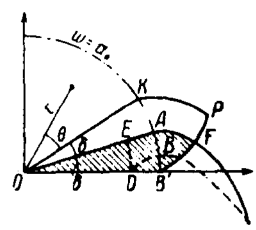

Fig. 2.

The boundary value problem corresponding to the given flow in the hodograph plane (Fig.2) can be formulated as follows:

The stream function $\psi(r, \theta)$ must be a solution of the Chaplygin equation [2]

$$\frac{\partial^2 \Psi}{\partial \tau^2} + \frac{1+(\beta-1)\tau}{\tau(1-\tau)}\frac{\partial \Psi}{\partial \tau} + \frac{1-\tau/\tau_s}{4\tau^2(1-\tau)}\frac{\partial^2 \Psi}{\partial \theta^2} = 0 \tag{1.1}$$

$$(\tau = w^2 / w^2_{max}) \quad \beta = (\varkappa - 1)^{-1}$$

which satisfies the following boundary conditions

$$\Psi(\tau, 0) = 0 \quad \text{for } 0 \leqslant \tau < \tau_0 \text{ (condition of symmetry)} \tag{1.2}$$

$$\Psi(\tau, \delta) = 0 \quad \text{for } 0 \leqslant \tau \leqslant \tau_s \text{ (condition of flow along the wall)} \tag{1.3}$$

$$X = l \quad \text{for } \theta = \delta \;\; \tau = \tau_s \text{ (condition that sonic velocity is attained at point } A) \tag{1.4}$$

$$\Psi = 0 \text{ on the characteristic } AF \text{ (condition at the center of expansion } A) \tag{1.5}$$

Here w is the fluid speed, w_{max} is the maximum speed, κ is the exponent of the adiabatic curve and $\tau_s = (2\beta+1)^{-1}$ corresponds to sonic velocity, while θ is the angle between the velocity vector and the x-axis. We must also satisfy the conditions on the shock wave bounding the supersonic zone *AB*. Further, since all stream lines in the hodograph plane originate from the same point $D(\tau_0, 0)$ corresponding to the flow velocity at infinity, the stream function at this point has a singularity. In the sonic case $(\tau_0 = \tau_s)$ the problem stated becomes a determinate triconic problem for $\psi(\tau, \theta)$ in the shaded region (Fig.2) of the (τ, θ) plane.

Previous investigations in this region have been confined to flows around thin wedges [3,4] with velocities near that of sound. Here all authors use Tricomi's equation [5]

$$\Psi_{\eta\eta} + \eta\Psi_{\theta\theta} = 0, \qquad \eta = \frac{(\varkappa+1)^{1/3}}{2}\left(1 - \frac{\tau}{\tau_s}\right) \tag{1.6}$$

Cole, in his paper [3] concerning the subsonic flow of a compressible gas around wedges, first found the solution to equation (1.6) satisfying all boundary conditions of the subsonic region (1.2), (1.3) and (1.4), and with the singularity corresponding to the flow at infinity. This solution cannot be considered completely satisfactory, however, since in the limiting case of sonic flow, shown in [6], the solution does not have the required singularity introduced by Frankl [1]. Trilling and Walker succeeded in satisfying condition (1.5) by adding to Cole's solution a series of regular solutions of (1.6). Using this more accurate solution the sonic line *AB* becomes curved (in comparison with Cole's sonic line). and more realistically shaped.

The problem of sonic flow about a wedge was solved by Guderley and Yoshihara [7] and Ovsiannikov [8] in the subsonic case, Tricomi's equation [1.6] was applied to investigate the flow.

In flow around a wedge, however, a critical point arises at the sharp nose in the vicinity of which velocities are quite small. It follows that equation (1.6) is not valid in this region and a more exact one (1.1) has to be applied. Mackie and Pack [9] have found the solution to this equation that satisfies conditions (1.2) and (1.3) and has the required singularity at this point $(\tau_0, 0)$. For this they used the method of generalizing an incompressible flow. Consider an incompressible flow past a profile with velocity at infinity equal to w_0 and having a complex potential

$$W_i = \sum d_\nu \left(\frac{w}{w_0}\right)^\nu e^{-i\nu\theta} \qquad (w < w_0) \tag{1.7}$$

The method of generalizing consists in changing from this potential to a "generalized" complex potential of the form

$$W = \Sigma\, d_\nu f_\nu(\tau_0)\, \psi_\nu(\tau)\, e^{-i\nu\theta} \qquad (\tau < \tau_0) \tag{1.8}$$

Here τ_0 corresponds to the oncoming stream, and
is Chaplygin's [2] function well known in gas dynamics.
F is a hypergeometric function with parameters

The "generalizing" function $f\nu(\tau_0)$ must satisfy the limiting condition $W \to W_i$ as τ, $\tau_0 \to 0(\beta \to 0)$ i.e.

$$f_\nu(\tau_0) \sim \tau_0^{-\nu/2} \qquad \text{as} \quad \tau_0 \to 0 \tag{1.9}$$

However, this condition is not sufficient to determine a unique "generalization". The major difficulty of the above method arises in the analytical continuation of series (1.8) into the region $\tau > \tau_0$ i.e. downstream beyond *EN*. Mackie and Pack [9] overcame this difficulty by using the simplest choice for $f\nu(\tau_0) = e^{-\nu s_0}$, where

$$\begin{gathered} S = \sigma + \tau_s^{-1/2} \operatorname{ar\,th}\left(\frac{\tau_s - \tau}{1-\tau}\right)^{1/2} - \operatorname{ar\,th}\left(\frac{1 - \tau/\tau_s}{1-\tau}\right)^{1/2} \\ \sigma = -\tau_s^{-1/2} \operatorname{ar\,th} \tau_s^{1/2} - \frac{1}{2}\ln\left(\frac{1}{2}\beta\right) \end{gathered} \tag{1.10}$$

Investigations of the above flow show that the corresponding stream function in the limiting case of sonic flow does not have the required singularity of Frankl [1]. Besides, the Mackie and Pack [9] generalized an incompressible flow, about a half-body consisting of a finite wedge of equal sides followed by a semi-infinite plate. We are interested in profiles of finite length.

In this work we initially formulate the problem of compressible flow around a finite double wedge-shaped profile which we shall solve by an approximate procedure based on the exact equation (1.1).

Consider the singularity corresponding to a stream at infinity with boundary conditions (1.2), (1.3) and (1.4). Here the use of the exact equation (1.1) permits us to consider a double wedge of arbitrary apex angle and arbitrary thickness. It is understood that the solution will not be unique, since the conditions in the supersonic region are not satisfied. To solve the problem we shall again use the method of generalizing an incompressible flow. In order to get rid of the major drawback of the solution of Mackie and Pack we shall impose new demands on the function $f_\nu(\tau_0)$ together with (1.9).

The generalizing function, while being as simple as possible, should give, in transonic flow, a stream function which has Frankl's singularity [1] i.e. has the form:

$$\Psi^*(\tau, \theta) \sim \Sigma \nu^{2/3} \frac{\psi_\nu(\tau)}{\psi_\nu(\tau_s)} \sin \nu\theta \tag{1.11}$$

Further, for nearly sonic values of τ_0 the function $f_\nu(\tau_0)$ must describe the "generalized" flow, corresponding to the transonic law of similarity [14]. With this we shall completely determine the method of approximating τ_0 to τ_s. The method described will then give correct flow behavior at the ends of the range of variation of $\tau_0[0, \tau_s]$. We should expect from this that the property of similarity will be fairly realistically defined for the functions $f_\nu(\tau_0)$ in the whole range of τ_0. It will be shown later that all the requirements imposed on $f_\nu(\tau_0)$ can be satisfied if the function is chosen to be of the following form:

$$f_\nu(\tau_0) = e^{-\nu s_0} \left\{1 + \frac{B\nu/\xi - 1}{1 + \zeta\nu}\right\} \qquad \left(\zeta = \frac{\sigma - s_0}{2\pi}\right) \tag{1.12}$$

where B is an arbitrary constant, and the factor $(2\pi)^{-1}$ is chosen for ease of calculation.

2. Subsonic gas flow past double wedge-shaped profile. The problem of incompressible fluid flowing symmetrically past a double wedge-shaped profile (Fig. 1), with the fluid velocity at infinity equal to w_0 can be solved with the help of conformal mapping.

Using the Schwarz-Christoffel theorem, we find the following complex potential

$$W_i = c^2 \left\{1 - \frac{1}{\sqrt{1 - \vartheta^\xi}}\right\} \qquad \left(\vartheta = \frac{w}{w_0} e^{-i\theta}\right)$$

Here c^2 is an arbitrary constant. Expanding this into a series, we get

$$W_i^{(1)} - = c^2 \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2^n n!} \vartheta^{n\xi} \qquad |\vartheta| < 1 \qquad (w < w_0) \tag{2.1}$$

The symbol $(2n-1)!!$ denotes the product of odd numbers. This series converges in the region $[\theta] = 1$, with the exception of point $\theta = 1$ which is a singular point and corresponds to the flow at infinity. Generalizing this result with use of (1.8) and (1.12) we obtain the following "generalized" complex potential

$$W^{(1)} = -c^2 \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2^n n!}\left(1 + \frac{Bn-1}{1+\zeta n\xi}\right) e^{-n\xi(s_0+i\theta)} \psi_{n\xi}(\tau) \tag{2.2}$$

and the stream function

$$\Psi^1(\tau, \theta) = c^2 \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2^n n!} \frac{B/\xi+\zeta}{1+\zeta n\xi} n\xi e^{-n\xi s_0} \psi_{n\xi}(\tau) \sin \tag{2.3}$$

These series converge for $r < r_9 (s < s_0)$. If we take account of the fact that as we get

$$f_\nu(\tau_0) \sim \tau_0^{-\nu/2}[1 + O(s_0^{-1})] \quad \text{for } \tau_0 \to 0,$$

i.e.

$$W^{(1)} \to W_i^{(1)} \text{ for } \tau_0 \to 0$$

and the requirement (1.9) is fulfilled. Cases of nearsonic and sonic $(r_0 = r_s)$ velocities of the oncoming stream will be given special consideration below. We shall note here however, that as shown in Section 3, (2.3) possesses Frankl' [1] singularity (1.11) at the point $(r_s, 0)$. Further, the constructed stream function satisfies the conditions (1.2) and (1.3) i.e. the axis of symmetry and the wall of the wedge act as one streamline. Further use of Chaplygin's equations [2]

$$\frac{\partial\varphi}{\partial\theta} = \frac{2\tau}{(1-\tau)^\beta}\frac{\partial\Psi}{\partial\tau}, \quad \frac{\partial\varphi}{\partial\tau} = -\frac{1-\tau/\tau_s}{2\tau(1-\tau)^{\beta+1}}\frac{\partial\Psi}{\partial\theta} \quad (\phi \text{ - velocity potential}) \tag{2.4}$$

and the transformation formula [2]

$$dx = \frac{\cos\theta}{w} d\varphi - \frac{\rho_0}{\rho}\frac{\sin\theta}{w} d\Psi$$

$$dy = \frac{\sin\theta}{w} d\varphi + \frac{\rho_0}{\rho}\frac{\cos\theta}{w} d\Psi \quad (\rho \text{ - density}) \tag{2.5}$$

along the symmetry axis $(\theta = 0)$ give

$$x\big|_{\theta=0} = -\frac{1}{w_{\max}} \int_0^\tau \frac{1-\tau/\tau_s}{2\tau(1-\tau)^{\beta+1}} \frac{\partial\Psi}{\partial\theta}\bigg|_{0=0} \frac{d\tau}{\sqrt{\tau}}$$

Since $0 < r < r_0$, using (2.3) and evaluating the integrals which appear as in Ref. [2], we shall obtain the following velocity distribution on the front face of the wedge.

$$x\big|_{\theta=0} = \frac{-c^2}{w_{\max}}\left(\frac{B}{\xi} + \frac{\sigma - s_0}{2\pi}\right) E(\tau) \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2^n n!} \frac{n^2\xi^2 e^{-n\xi s_0}}{n\xi^2 - 1} \frac{G_{n\xi}(\tau)}{1+\zeta n\xi} \tag{2.6}$$

$$\left(E(\tau) = \frac{1}{\sqrt{\tau}(1-\tau)^\beta}, \qquad G_\nu(\tau) = \psi_\nu(\tau) + 2\tau\psi_\nu'(\tau)\right)$$

To show that $x(r_0) \to -\infty$ for $\theta \to 0$ we shall turn to the asymptotic behavior of the Chaplygin functions. Lighthill obtained the following formula

$$\psi_\nu(\tau)=\left[\frac{(1-\tau)^{2\beta+1}}{1-\tau/\tau_s}\right]^{1/4}e^{\nu s}\left\{1+O\left(\frac{1}{|\nu|}\right)\right\} \tag{2.7}$$

which is valid for $|\nu|\to\infty$, $0<r<r_s$ and for the whole complex plane ν, excluding circles of arbitrary small radii with centers $\nu=-n$, $(n=2, 3, 4 \ldots)$. For the same r and real ν we [16] found the asymptotic relation

$$2\tau\psi_\nu'(\tau)=\nu\left(1-\frac{\tau}{\tau_s}\right)^{1/4}(1-\tau)^{\beta/2-1/4}e^{\nu s}\left\{1+O\left(\frac{1}{|\nu|}\right)\right\} \tag{2.8}$$

Further, applying Stirling's formula [10] we have $(n\to\infty)$

$$\frac{(2n-1)!!}{2^n n!}=\frac{1}{\sqrt{\pi n}}\left[1+O\left(\frac{1}{n}\right)\right] \tag{2.9}$$

Substitution of the values found into (2.6) will lead to a divergent series, as expected. Hence, $x(r)\to-\infty$ as $\theta\to 0$ and (2.3) determines a physically feasible flow. It remains to connect the xy and $r\,\theta$ planes by fulfilling condition (1.4), which determined c^2. For this we have to know the stream function $\psi(r, \theta)$ in the region $[0, r_s]$. But the point $r=r_0$, $\theta=0$ is singular. As a result of this the region of convergence of series (2.3) is limited to the area $r=r_0$ in the hodograph plane or the curve EN in the physical plane. Therefore it is required to continue the series (2.3) analytically through the region $r=r_0$. This constitutes the major difficulty of the method applied.

3. Analytical continuation of series (2.3) into region $r>r_0$. Analytical continuation is performed by a proper choice of an auxiliary Barnes' integral [10]. Consider

$$W=\frac{c^2}{2\pi^{3/2}i}\left(\frac{B}{\xi}+\frac{\sigma-s_0}{2\pi}\right)\int_{-i\infty}^{+i\infty}\Gamma\left(1-\frac{\nu}{\xi}\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{\xi}\right)\times$$
$$\times\frac{\exp\left[-\nu(s_0+i\theta)+i\nu\delta\right]}{1+\zeta\nu}\Psi_\nu(\tau)\,d\nu \tag{3.1}$$

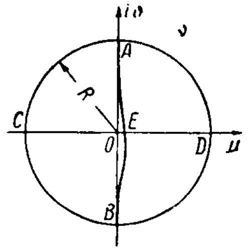

Fig. 3.

The path of integration AEB is chosen so that simple poles $n\xi$, $(n = 1, 2 \ldots)$ are located to the right of this path and all remaining poles to the left. In order to exclude the possibility of appearance of second-order poles, we shall assume ξ to be irrational, and such that $\zeta = (\sigma - s_0)/2\pi$ will not coincide with $n + 1$ and $(n - 1/2)\xi$. Then to the left of AEB, the expression under the integral (3.1) $J(\nu, \theta, \tau)$ has simple poles at the points $\nu = -(n + 1)$, $\nu = -(n - 1/2)\xi$ and $\nu = -2\pi/(\sigma - s_0)$. Let us show that (3.1) for $\tau < \tau_0$ transforms into (2.2). For this we shall close the path AEB by a semicircle ADB of radius $R = N\xi + 1/2$, located to the right of the imaginary axis, which does not contain any singular points when N is an integer (Fig.3). Evaluating $J(\nu, \theta, \tau)$ on the semicircle ADB of a large radius $|\nu| = R \to \infty$ with the aid of asymptotic gamma-functions [10] and (2.7) we obtain

$$|J(\nu, \theta, \tau)| \sim \frac{4\pi^2}{\sigma - s_0}(R\xi)^{-1/2} \exp\{\mu(s - s_0) - \vartheta(\delta \pm \delta - \theta)\}$$

where we have a minus sign for $\theta < 0$ and a plus sign for $\theta > 0$.

Hence the integral (3.1) approaches zero as $R \to \infty$ and $\tau < \tau_0$, $0 < \theta < 2\gamma$. Therefore, the residue theorem [10] applied to the region $AEBDA$ gives

$$W = c^2 \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2^n n!}\left(1 + \frac{Bn - 1}{1 + \zeta n \xi}\right) \exp\{-n\xi(s_0 + i\theta)\}\, \psi_{n\xi}(\tau) = W^{(1)}$$

The series obtained converges for any θ and $\tau < \tau_0$.

On the opposite side we shall complete the path of integration AEB by a semi-circle ACB of the same radius, located to the left of the imaginary axis. Then by analogy we can show that integral (3.1) vanishes on the curve ACB as $R \to \infty$ for $\tau_0 < \tau < \tau_s$ and $0 < \theta < 2\delta$ and the residue theorem leads to the following:

$$W^{(2)} = -c^2 \Big\{ i \sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n n!} \frac{(n+1/2)\psi_{-(n+1/2)\xi}(\tau)}{1 - (n+1/2)\xi/z} \exp\left[\left(n + \tfrac{1}{2}\right)\xi(s_0 + i\theta)\right] +$$

$$+ \frac{1}{\xi\sqrt{\pi}} \sum_{n=2}^{\infty} \Gamma\left(1 + \frac{n}{\xi}\right)\Gamma\left(\frac{1}{2} - \frac{n}{\xi}\right) \frac{nC_n\psi_n(\tau)}{1 - n/z} \exp\left[n(s_0 + i(\theta - \delta))\right]\Big\}\left(B + \frac{\xi}{z}\right) -$$

$$- \frac{1}{\sqrt{\pi}} \Gamma\left(1 + \frac{z}{\xi}\right)\Gamma\left(\frac{1}{2} - \frac{z}{\xi}\right)\frac{z}{\xi} \exp\left[s_0 + i(\delta - \theta)\right] z\psi_{-z}(\tau)\Big\}$$

Here* $\qquad \dfrac{(2n-1)!!}{2^n n!} = 1 \quad$ for $n = 0, \qquad \dfrac{2\pi}{\sigma - s_0} = z$

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*The residue $\psi_\nu(\tau)$ at pole $\nu = -n$ is equal to $-nc_n\psi_n(\tau)$ where

$$C_n = \frac{\Gamma(a_n)\Gamma(n - b_n + 1)}{\Gamma(a_n - n)\Gamma(1 - b_n)\Gamma^2(n+1)}$$

It follows that the stream function for $r > r_0$ has the form:

$$\psi^{(2)}(\tau, \theta) = -c^2\left(B + \frac{\xi}{z}\right)\left\{\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n n!} \frac{(n+1/2)}{1-(n+1/2)\xi/z} \exp\left[\left(n+\frac{1}{2}\right)\frac{\xi}{z}\right] \times\right.$$

$$\times \psi_{-(n+1/2)\xi}(\tau)\cos\left(n+\frac{1}{2}\right)\xi\theta - \frac{1}{\xi\sqrt{\pi}}\sum_{n=2}^{\infty}\Gamma\left(1+\frac{n}{\xi}\right)\Gamma\left(\frac{1}{2}-\frac{n}{\xi}\right)\frac{nC_n}{1-n/z}e^{ns_0}\times$$

$$\times \psi_n(\tau)\sin n(\delta-\theta) - 2\sqrt{\pi}\,\Gamma\left(1+\frac{z}{\xi}\right)\Gamma\left(\frac{1}{2}-\frac{z}{\xi}\right)\frac{\exp(zs_0)}{(\sigma-s_0)\xi}\psi_{-z}(\tau)\times$$

$$\left.\times\sin\left(2\pi\frac{\delta-\theta}{\sigma-s_0}\right)\right\} \qquad (3.2)$$

and, as can be seen, immediately satisfies the condition (1.3). Besides, the series converges for any θ and $r_0 < r < r_s$. In the particular case $r_0 = r_s$ using the asymptotic form [18] we can show that the integral (3.1) is defined in the characteristic triangle *KPB′* with arc 2δ (Fig.2).

To find the velocity distribution on the front face of the wedge we shall integrate (2.5) along $\theta = \delta$. After substituting (2.3) and (3.2) we have for $0 < r < r_0$

$$x^1(\tau, \tau_0) = \frac{c^2}{w_{\max}}\cos\delta\left(B+\frac{\xi}{z}\right)E(\tau)\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n n!}\frac{n^2\xi e^{-n\xi s_0}}{n^2\xi^2-1}\frac{G_{n\xi}(\tau)}{1+n\xi/z}$$

для $\tau_0 \leqslant \tau \leqslant \tau_s$

$$x^{(2)}(\tau, \tau_0) = \frac{c^2}{w_{\max}}\cos\delta\left(B+\frac{\xi}{z}\right)\left\{\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n n!}\frac{1}{(n+1/2)^2\xi^2-1}\times\right.$$

$$\times\frac{(n+1/2)^2\xi\exp[(n+1/2)\xi]/s_0}{1-(n+1/2)\xi/z}\left[E(\tau)G_{-(n+1/2)\xi}(\tau) - E(\tau_0)G_{-(n+1/2)}(\tau_0)\right] +$$

$$+\frac{1}{\xi\sqrt{\pi}}\sum_{n=2}^{\infty}\Gamma\left(1+\frac{n}{\xi}\right)\Gamma\left(\frac{1}{2}-\frac{n}{\xi}\right)\frac{n^2C_n e^{ns_0}}{(1-n/z)(n^2-1)}\left[E(\tau)G_n(\tau)-E(\tau_0)G_n(\tau_0)\right] -$$

$$-\frac{1}{\xi\sqrt{\pi}}\Gamma\left[1+\frac{2\pi}{\xi(\sigma-s_0)}\right]\Gamma\left[\frac{1}{2}-\frac{z}{\xi}\right]\frac{e^{zs_0}}{1-z^{-2}}\left[E(\tau)G_{-z}(\tau)-E(\tau_0)\dot{G}_{-z}(\tau_0)\right]\Big\} +$$

$$+ x^{(1)}(\tau_0, \tau_0) \qquad (3.3)$$

Hence, satisfying the condition (1.4) we obtain the equation $x^{(2)}(r_s, r_0) = l$ for the determination of c^2. Summing, we find the coefficient of local pressure

$$C_p = \frac{p-p_1}{1/2\rho_1 w_0^2} = \frac{1-\tau_0}{(\beta+1)\tau_0}\left\{\left(\frac{1-\tau}{1-\tau_0}\right)^{\beta+1} - 1\right\}$$

(p_1, ρ_1 are the pressure and density of the oncoming flow).

Along the face of the wedge *CA* we find the coefficient of frontal pressure per unit span:

$$C_x = \frac{\operatorname{tg}\delta}{l}\left[\int\limits_0^{\tau_0} C_p \frac{dx^{(1)}}{d\tau}\,d\tau + \int\limits_{\tau_0}^{\tau_s} C_p \frac{dx^{(2)}}{d\tau}\,d\tau\right]_{\theta=\pi/\xi}$$

The integration of the latter by parts and substitution of (3.3) give

$$C_x = \frac{\varkappa - 1}{\varkappa}\operatorname{tg}\delta\,\frac{1-\tau_0}{\tau_0}\left\{\left(\frac{1-\tau_s}{1-\tau_0}\right)^{\beta+1} - 1\right\} +$$

$$+ \frac{c^2}{w_{\max}}\left(B + \frac{\xi}{z}\right)\frac{2\sin\delta}{l\tau_0(1-\tau_0)^\beta}\left\{\sqrt{\tau_0}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n n!}\,\frac{n\xi e^{-n\xi s_0}}{n^2\xi^2-1}\,\frac{n\psi_{n\xi}(\tau_0)}{1+n\xi/z} +\right.$$

$$+ \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n n!}\,\frac{(n+{}^1/_2)^2\,\xi e^{(n+{}^1/_2)\xi s_0}}{(n+{}^1/_2)^2\xi^2-1}\,\frac{1}{1-z^{-1}(n+{}^1/_2)\xi}\Big[\sqrt{\tau_s}\,\psi_{-(n+{}^1/_2)\xi}(\tau_s) -$$

$$- \sqrt{\tau_0}\,\psi_{-(n+{}^1/_2)\xi}(\tau_0) + \frac{E(\tau_0)}{2(\beta+1)}G_{-(n+{}^1/_2)\xi}(\tau_0)\left((1-\tau_s)^{\beta+1} - (1-\tau_0)^{\beta+1}\right)\Big] +$$

$$+ \frac{1}{\xi\sqrt{\pi}}\sum_{n=2}^{\infty}\Gamma\left(1+\frac{n}{\xi}\right)\Gamma\left(\frac{1}{2}-\frac{n}{\xi}\right)\frac{n^2 C_n e^{n s_0}}{(n^2-1)(1-n/z)}\Big[\sqrt{\tau_s}\,\psi_n(\tau_s) - \sqrt{\tau_0}\,\psi_n(\tau_0) +$$

$$+ \frac{E(\tau_0)}{2(\beta+1)}G_n(\tau_0)\left((1-\tau_s)^{\beta+1} - (1-\tau_0)^{\beta+1}\right)\Big] -$$

$$- \frac{1}{\xi\sqrt{\pi}}\Gamma\left[1+\frac{2\pi}{(\sigma-s_0)\xi}\right]\Gamma\left[\frac{1}{2}-\frac{2\pi}{(\sigma-s_0)\xi}\right]\frac{e^{z s_0}}{1-z^{-2}}\Big[\sqrt{\tau_s}\,\psi_{-z}(\tau_s) - \sqrt{\tau_0}\,\psi_{-z}(\tau_0) +$$

$$\left.+ \frac{E(\tau_0)}{2(\beta+1)}G_{-z}(\tau_0)\left((1-\tau_s)^{\beta+1} - (1-\tau_0)^{\beta+1}\right)\Big]\right\}. \qquad (3.4)$$

Equations (3.3) has been evaluated [19, 20] for $\xi = 11.6129$ and $\tau = 0.8$, 0.12, corresponding to a wedge apex angle $2\gamma = 31^\circ$ and free stream Mach number $M_0 = 0.660$, 0.826. Results are plotted in Fig. 4.

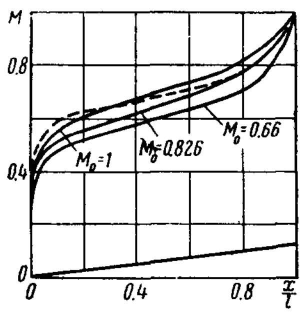

Fig. 4.

4. Flow of a sonic stream past a double wedge. In the limiting case, when $\tau_0 = \tau_s (s_0 = \sigma)$ the stream function (2.3) has the form

$$\psi^{*}(\tau,\theta)=c^{2}B\sum_{n=1}^{\infty}\frac{(2n-1)!!}{2^{n}n!}ne^{-n\xi\sigma}\psi_{n\xi}(\tau)\sin(n\xi\theta) \tag{4.1}$$

This series converges absolutely and uniformly for $0 < \tau < \tau_s$ and arbitrary θ and has a singularity at the point $\tau = \tau_s$, $\theta = 0$.

In order to explain its characteristics, let us perform the following identical transformation:

$$\Psi^{*}(\tau,\theta)=c^{2}B\sum_{n=1}^{\infty}\left\{\frac{(2n-1)!!}{2^{n}n!}ne^{-n\xi\sigma}\psi_{n\xi}(\tau)-\frac{\alpha\xi^{1/6}}{\sqrt{\pi}}n^{1/3}\frac{\psi_{n\xi}(\tau)}{\psi_{n\xi}(\tau_s)}\right\}\times$$
$$\times\sin(n\xi\theta)+c^{2}B\xi^{1/6}\alpha\sum_{n=1}^{\infty}n^{1/3}\times\frac{\psi_{n\xi}(\tau)}{\psi_{n\xi}(\tau_s)}\sin(n\xi\theta)$$

Here
$$\alpha=2^{\frac{2\varkappa-1}{2(\varkappa-1)}}(\varkappa+1)^{-\frac{\varkappa+2}{6(\varkappa-1)}}\pi^{\frac{1}{2}}3^{-\frac{2}{3}}\Gamma^{-1}\left(\frac{2}{3}\right) \tag{4.2}$$

is the coefficient in the asymptotic formula ($\nu \to +\infty$) derived by us [16] which for transonic velocities τ can be expressed as

$$\psi_{\nu}(\tau)=\alpha\nu^{1/6}e^{\nu\sigma}\frac{v(\nu^{2/3}\eta)}{v(0)},\quad 2\tau\psi_{\nu}'(\tau)=-\alpha(\varkappa+1)^{1/3}\nu^{5/6}e^{\nu\sigma}\frac{v'(\nu^{2/3}\eta)}{v(0)}$$
$$\psi_{-\nu}(\tau)=\alpha\nu^{1/6}e^{-\nu\sigma}\left\{\frac{1}{2v^{2}(0)}[v(0)u(\nu^{2/3}\eta)-u(0)v(\nu^{2/3}\eta)]+\frac{v(\nu^{2/3}\eta)}{2v(0)}(\sqrt{3}+\operatorname{ctg}\nu\pi)\right\}$$
$$2\tau\psi_{-\nu}'(\tau)=-\alpha(\varkappa+1)^{1/3}\nu^{5/6}e^{-\nu\sigma}\left\{\frac{1}{2v^{2}(0)}[v(0)u'(\nu^{2/3}\eta)-u(0)v(\nu^{2/3}\eta)]+\right.$$
$$\left.+\frac{v'(\nu^{2/3}\eta)}{2v(0)}(\sqrt{3}+\operatorname{ctg}\nu\pi)\right\} \tag{4.3}$$

Here $u(s)$ and $v(s)$ are two linearly independent solutions of Airy's equations $U''(s) - sV(s) = 0$ tabulated by Fok [11]. Using (2.9) the convergence of the first series (4.2) at $\tau = \tau_s$ becomes obvious. From here it follows that the important singularity is completely represented by the second series, which coincides with the main term of Frankl's [1] singular solution (1.11). Even though in the approximate formulation of the problem conditions in the supersonic flow region remain unsatisfied, the singularity, corresponding to an incident sonic stream, turns out to be chosen correctly. Therefore [1] the stream function (4.1) is regular on the characteristics $B'F$ (Fig.2) and determines the flow where all the stream lines cross the sonic line. Assuming that $\tau = \tau_0$ in $x^{(1)}(\tau, \tau_0)$ given by (3.3), ($s_0 = \sigma$) we shall obtain the formula for the velocity distribution on the front face of the wedge. In this case, condition (1.4), which defines the scale coefficient c^2B, should be taken in its limiting form

$$\left(\frac{c^2B\cos\delta}{w_{\max}}\right)^{-1} = \frac{E(\tau_s)}{l}\lim_{\varepsilon\to 0}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n\, n!}\,\frac{n^2\xi}{n^2\xi^2-1}\,e^{-n\xi\sigma}G_{n\xi}(\tau_s-\varepsilon)$$

In order to realize the transition to the limit, we transform this to

$$\left(\frac{c^2B\cos\delta}{w_{\max}}\right)^{-1} = \frac{E(\tau_s)}{l}\lim_{\varepsilon\to 0}\sum_{n=1}^{\infty}(-1)^{n+1}\left\{\frac{(2n-1)!!}{2^n\, n!}\,\frac{n^2\xi}{n^2\xi^2-1}\,e^{-n\xi\sigma}\times\right.$$
$$\left.\times\, G_{n\xi}(\tau_s-\varepsilon) + \frac{\alpha(\varkappa+1)^{1/3}n^{1/3}}{\xi^{1/6}\pi^{1/2}V(0)}\,v\left[n^{2/3}\xi^{2/3}\frac{(\varkappa+1)^{4/3}}{2(\varkappa-1)}\,\varepsilon\right]\right\} -$$
$$-\frac{\alpha E(\tau_s)(\varkappa+1)^{1/3}}{l\sqrt{\pi}\,\xi^{1/6}v(0)}\lim_{\eta\to 0}\sum_{n=1}^{\infty}(-1)^{n+1}n^{1/3}v'(n^{2/3}\xi^{2/3}\eta) \qquad (4.4)$$

By applying (2.9) and (4.3) we can show that it is permissible to interchange the order of the operations lim and Σ in the first series. The last limit was previously calculated [3] as

$$\lim_{\eta\to 0}\sum_{n=1}^{\infty}(-1)^{n+1}n^{1/3}v'(n^{2/3}\xi^{2/3}\eta) = -v'(0)(2^{4/3}-1)\,\zeta(-\tfrac{1}{3}) \qquad (4.5)$$

where ζ is the zeta function [10]. Hence (4.4) and (4.5) determine c^2B. The coefficient of frontal pressure is determined from (3.4) by substituting $r_0 = r_s$. The velocity distribution on the wedge $\xi = 11.6129$ ($\delta \approx 15°5$) in the case of sonic flow is shown in Fig. 7. Here we also show the results of Ovsiannikov [8] for a wedge with $\delta = 15°$, using a dotted line.

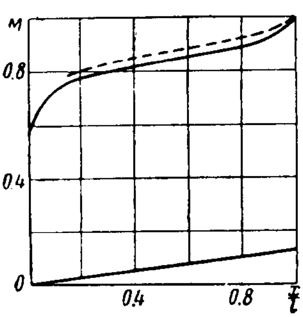

Fig. 5.

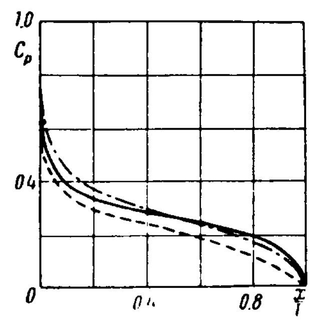

Fig. 6.

If we consider the flow past a thin wedge, then ξ is large and Chaplygin's functions can be replaced by their asymptotic representations (4.3) i.e.

$$x^*(\tau) = \frac{c^2B}{w_{\max}}\frac{E(\tau)}{\xi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n n!}e^{-n\xi\sigma}G_{n\xi}(\tau) \tag{4.6}$$

$$C_x^* = \left[\frac{c^2B\alpha(\varkappa+1)^{1/3}}{w_{\max}\xi^{1/3}l\sqrt{\tau_s}(1-\tau_s)}\right]\left[\frac{\delta^{5/3}}{(\varkappa+1)^{1/3}}\right]\frac{2}{\pi^{1/2}}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{1/3}}\frac{(2n-1)!!}{2^n n!} \tag{4.7}$$

where

$$\left[\frac{c^2B\alpha(\varkappa+1)^{1/3}}{w_{\max}\xi^{1/3}l\sqrt{\tau_s}(1-\tau_s)}\right]^{-1} = -\frac{v'(0)}{v(0)}\left\{\sum_{n=1}^{\infty}\left[\frac{(2n-1)!!}{2^n n!}n^{1/3}-\frac{n^{1/3}}{\sqrt{\pi}}\right](-1)^{n+1}+\right.$$

$$\left.+\frac{2^{4/3}-1}{\sqrt{\pi}}\zeta\left(-\frac{1}{3}\right)\right\}$$

The graphs of the velocity and pressure distribution on a wedge with a 10 per cent relative thickness are given in Fig. 5 and 6 respectively. For comparison we show experimental results [17] as the dotted curve. In Fig. 6 we also show the results of the theoretical investigations of Guderley [7] for a wedge of the same thickness. For C_x^* we shall note that (4.7) gives

$$C_x^* = 1.77\,\frac{\delta^{5/3}}{(\varkappa+1)^{1}} \qquad \left(C_x^* = 1.75\,\frac{\delta^{5/3}}{(\varkappa+1)^{1/3}}\right)$$

The latter (in brackets) are the results of Guderley [7].

5. Transonic gas flow past a thin double wedge. In order to obtain the case of transonic flow past a thin double wedge from the general results of Section 3, we assume that ξ^{-1} and $r_s - r_0$ are small. We can then apply the asymptotic formula (4.3) in (1.4) and (3.4). We combine the velocity of the oncoming stream $r_0(s_0)$ and the angle of the wedge $\delta(\xi)$ to form a parameter of transonic similarity

$$k_0 = \frac{1-M_0^2}{[(\varkappa+1)\delta]^{2/3}}$$

Furthermore, in order that the coefficient of frontal pressure corresponding to the law of transonic similarity shall have the form:

$$C_x = \frac{\delta^{5/3}}{(\varkappa+1)^{1/3}}f(k_0)$$

we shall disregard the following terms in the asymptotic expansion (4.3)

$$\operatorname{ctg}(n+\tfrac{1}{2})\xi\pi, \qquad \operatorname{ctg}\frac{2\pi^2}{\sigma-s_0}$$

Then to the order of ξ^{-1} and $r_s - r_0$ we shall have

$$C_x=\left\{\frac{c^2(B+{}^1/_3k_0{}^{3/2})\,\alpha\,(\varkappa+1)^{1/3}}{w_{\max}\,l\xi^{1/6}\sqrt{\tau_s}\,(1-\tau_s)^{\beta}}\right\}\frac{\delta^{5/3}}{(\varkappa+1)^{1/3}}\left\{\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n\,n!}n^{1/6}\times\right.$$

$$\times\frac{1}{1+{}^1/_3nk_0{}^{3/2}}\exp\left(\frac{2}{3}n\pi k_0{}^{3/2}\right)v\,(n^{2/3}\pi^{2/3}k_0)+\frac{1}{2v\,(0)}\sum_{n=0}^{\infty}(-1)\frac{(2n-1)!!}{2^n\,n!}\left(n+\frac{1}{2}\right)^{1/6}\times$$

$$\times\frac{\exp\left[-{}^2/_3(n+{}^1/_2)\,\pi k_0{}^{3/2}\right]}{1-{}^1/_3\,(n+{}^1/_2)\,k_0{}^{3/2}}\left[\sqrt{3}\,v\,(0)-\frac{1}{v\,(0)}\left[v\,(0)\,u\,\{(n+{}^1/_2)^{2/3}\pi^{2/3}k_0\}-\right.\right.$$

$$\left.-u\,(0)\,v\,\{(n+\tfrac{1}{2})^{2/3}\pi^{2/3}k_0\}\right]-\sqrt{3}\,v\,\{(n+\tfrac{1}{2})^{2/3}\pi^{2/3}k_0\}]-\frac{3^{5/6}e^{-2\pi}}{2\sqrt{\pi}\,v\,(0)\,k_0{}^{1/4}}\times$$

$$\times\Gamma\left(1+\frac{3}{k_0{}^{3/2}}\right)\Gamma\left(\frac{1}{2}-\frac{3}{k_0{}^{3/2}}\right)\left[\sqrt{3}\,v\,(0)-\frac{1}{v\,(0)}\left[v\,(0)\,u\,\{(3\pi)^{2/3}\}-u\,(0)\,u\,\{(3\pi)^{2/3}\}\right]-\right.$$

$$\left.-\sqrt{3}v\,\{(3\pi^2)^{2/3}\}\right]\frac{2}{\pi^{2/3}v(0)}$$

Where

$$\left\{\frac{c^2\,(B+{}^1/_3\,k_0{}^{3/2})\,\alpha\,(\varkappa+1)^{1/3}}{w_{\max}\,l\xi^{1/6}\sqrt{\tau_s}\,(1-\tau_s)^{\beta}}\right\}^{-1}=\frac{-1}{v\,(0)}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n\,n!}v'\,(n^{2/3}\,\pi^{2/3}\,k_0)\times$$

$$\times\frac{n^{5/6}\exp\,({}^2/_3\,n\pi k_0{}^{3/2})}{1+{}^1/_3\,nk_0{}^{3/2}}-\frac{1}{2v\,(0)}\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{2^n\,n!}\frac{(n+{}^1/_2)^{5/6}}{1-{}^1/_3\,(n+{}^1/_2)\,k_0{}^{3/2}}\times$$

$$\times\exp\left[-\frac{2}{3}\left(n+\frac{1}{2}\right)\pi k_0{}^{3/2}\left\{-\frac{1}{v\,(0)}\left[v\,(0)\,u'\left[\left(n+\frac{1}{2}\right)^{2/3}\pi^{2/3}k_0\right]-u\,(0)\times\right.\right.\right.$$

$$\left.\left.\times\,v'\left[\left(n+\frac{1}{2}\right)^{2/3}\pi^{2/3}k_0\right]\right]+v'\,(0)\,\sqrt{3}+\frac{1}{v\,(0)}-\sqrt{3}v'\left[\left(n+\frac{1}{2}\right)^{2/3}\pi^{2/3}k_0\right]\right\}+$$

$$+\frac{3^{5/6}e^{-2\pi}}{2\sqrt{\pi}\,v\,(0)\,k_0{}^{5/4}}\,\Gamma\,(1+3k_0{}^{-3/2})\,\Gamma\,(\tfrac{1}{2}-3k_0{}^{-3/2})\left\{V'\,(0)\sqrt{3}+\frac{1}{v\,(0)}-\right.$$

$$\left.-\sqrt{3}v'\,[(3\pi)^{2/3}]-\frac{1}{v\,(0)}\,[v\,(0)\,u'\,[(3\pi)^{2/3}]-u\,(0)\,v'\,[(3\pi)^{2/3}]]\right\}$$

In Fig. 4 we have the calculated velocity distribution on a wedge with $\delta = 7^{\circ}.5$. For purposes of comparison we have plotted experimental curves [12] for a wedge of $\delta = 7^{\circ}.5$ followed by a semi-infinite plate. They are shown as dotted lines.

Fig. 8 shows $C_x^0 = C_x(\kappa + 1)^{1/3}/\delta^{5/3}$ as a function of k_0 and experimental points [12]. Using some cumbersome expressions, we have also shown that on the surface of the wedge the above $\delta^{5/3}$ formulas give

$$\left.\frac{dC_x}{dk_0}\right|_{k_0=0}=-2\,\frac{\delta^{5/3}}{(\varkappa+1)^{1/3}},\qquad\left.\frac{dM}{dM_0}\right|_{M_0=1}=0$$

where M is a local Mach number. This fact was established experimentally [12,17].

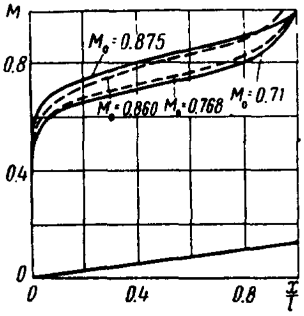

Fig. 7.

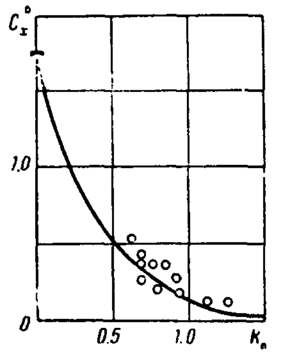

Fig. 8.

In the case of transonic flow, we are especially interested in the form of the sonic line. The latter allows us to estimate the size and the behavior of the local supersonic zone near point A (Fig. 1). Integrating (2.5) along $\tau = \tau_s$, we find, to the order of ξ^{-1}, that the coordinates of the transition line in a system with origin at point A are

$$y_s = \frac{1}{a_* (1-\tau_s)^\beta} \Psi^{(2)}(\tau_s, \theta), \qquad x_s = \frac{2\tau_s}{a_* (1-\tau_s)^\beta} \int_\delta^\theta \frac{\partial \Psi^{(2)}}{\partial \tau}\bigg|_{\tau=\tau_s} d\theta \quad (5.1)$$

where a_* is the critical sound velocity. Within the limits of the accepted approximation (2.11) has the form

$$\Psi^{(2)}(\tau, \theta) = -c^2 \left(B + \frac{1}{3} k_0^{3/2}\right)\left\{\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n n!} \frac{n + 1/2}{1 - 1/3\,(n + 1/2)\, k_0^{3/2}} e^{(n+1/2)\,\xi s_0} \times \right.$$
$$\times \Psi_{-(n+1/2)\,\xi}(\tau) \cos\left(n + \frac{1}{2}\right)\xi\theta - \frac{3}{\sqrt{\pi}\, k_0^{3/2}} \Gamma\left(1 + \frac{3}{k_0^{3/2}}\right)\Gamma\left(\frac{1}{2} - \frac{3}{k_0^{3/2}}\right) \times$$
$$\left. \times\, e^{zS_0} \psi_{-z}(\tau) \sin\left[\frac{3}{k_0^{3/2}}(\pi - \xi\theta)\right]\right\}$$

When introducing the latter into (5.1) and applying the asymptotic (4.3) it is necessary to disregard the terms with $\mathrm{ctg}(n + 1/2)\,\xi\pi$ and $\mathrm{ctg}[2\pi^2/(\sigma - s_0)]$ so as not to change the law of transonic similarity [4]. As a result we obtain the equation of the sonic line in the parametric form

$$Y = y_s(\varkappa + 1)^{1/3}\delta^{1/3} = -\frac{c^2 (B + 1/3\, k_0^{3/2})\, a\pi^{1/6}\sqrt{3}}{2a_* \xi^{1/6} (\varkappa + 1)^{-1/3} (1 - \tau_s)^\beta}\left\{\sum_{n=0}^{\infty} \frac{(2\pi - 1)!!}{2^n n!} \times \right.$$
$$\times \exp\left[-\frac{2}{3}\left(n + \frac{1}{2}\right)\pi k^{3/2}\right] \times \frac{(n + 1/2)^{7/6}}{1 - 1/3\,(n + 1/2)\, k_0^{3/2}} \cos\left(n + \frac{1}{2}\right)\xi\theta +$$
$$\left. + \frac{3^{7/6} e^{-2\pi}}{\sqrt{\pi}\, k_0^{7/4}} \Gamma\left(1 + \frac{k_0^{3/2}}{3}\right)\Gamma\left(\frac{1}{2} - \frac{1}{3} k_0^{3/2}\right) \sin\left[\frac{3}{k_0^{3/2}}\left(1 - \frac{\theta}{\delta}\right)\right]\right\}$$

$$x_s = \frac{c^2 (B + {}^1/_3 K_0^{3/2})\,\alpha\,(\varkappa + 1)^{1/3}\,[\sqrt{3}v'(0) + 1/v(0)]}{2a\xi^{2/3}\,v(0)\,(1-\tau_s)^{\beta}} \left\{ \sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n n!} \left(n+\frac{1}{2}\right)^{5/6} \times \right.$$

$$\times \frac{\exp[-{}^2/_3 (n + {}^1/_2)\,\pi k_0^{3/2}]}{1 - {}^1/_3 (n + {}^1/_2)\, k_0^{3/2}} \left[(-1)^{n+1} + \sin\left(n+\frac{1}{2}\right)\pi\frac{\theta}{\delta} \right] +$$

$$\left. + \frac{3^{5/6} e^{-2\pi}}{\sqrt{\pi}\, k_0^{5/4}} \Gamma(1 + 3k_0^{-3/2})\, \Gamma\left(\frac{1}{2} - 3k_0^{-3/2}\right) \left[\cos\frac{3\pi}{k_0^{3/2}} \left(1 - \frac{\theta}{\delta}\right) - 1 \right] \right\}$$

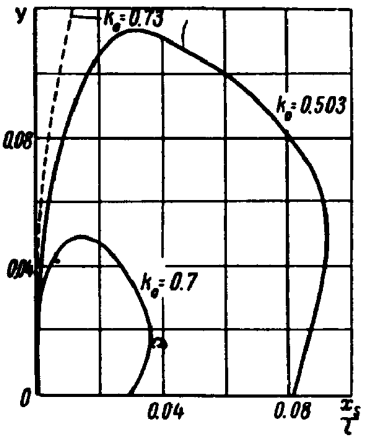

Fig. 9.

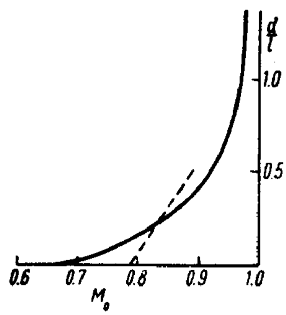

Fig. 10.

The curves obtained are plotted in Fig. 9 where the dotted line shows the experimental [12] results for a wedge with a semi-infinite plate downstream. It is understood that the disturbance decreases as we get away from the profile, so that $d = \max$, $y_s = y_s(\min\theta)$ determines exactly the diameter of the local supersonic region in terms of k_0. In Fig. 10 the result for a double wedge of 12 per cent relative thickness and the experimental [15] result (dotted) for an airfoil section of the same thickness are compared. Here we can see directly that the supersonic region is enlarged with increase of velocity of the oncoming stream and extends to infinity when $M_0 = 1$. It also follows, from the character of Frankl's singularity, that in this case the sonic line away from the profile has the form

$$y_s = Cx_s^{3/4}$$

From this we can deduce that the "generalizing function" (1.12) not only gives a correct qualitative picture of the flow past double wedge in the region $0 < M_0 < 1$ but also leads us to quantitative results which are in a complete agreement with the experimental ones. Naturally we cannot expect a unique choice of $f_{\nu}(\tau_0)$, as the problem was solved in an approximate form and the conditions in the supersonic region remained unsatisfied. However, when the velocity of the oncoming stream is not great, the supersonic region is small and its influence can be neglected. As we approach the sonic flow about a double wedge our solution is still correct and describes the existing flow with an adequate degree of accuracy because of the accurate character of the singularity, and of the stream function for this flow. Besides, as shown in [13], the satisfaction of condition 1.5, governing the expansion of the stream at the point A (Fig. 1) leads to only negligible changes of the aerodynamic characteristics of the flow.

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